Drop-Tower Experiments for Capillary Surfaces in an Exotic Container

Paul Concus*
University of California, Berkeley, Berkeley, California 94720
Robert Finn†
Stanford University, Stanford, California 94305
and
Mark Weislogel‡
NASA Lewis Research Center, Cleveland, Ohio 44135

Low-gravity drop-tower experiments were carried out for an "exotic" rotationally symmetric container, which admits an entire continuum of distinct equilibrium symmetric capillary free surfaces. It was found that an initial equilibrium planar interface, a member of the continuum, can reorient toward a nonsymmetric interface, as predicted by recent mathematical theory.

Introduction

GENERALLY, the free surface of a liquid that partly fills a container under the action of surface and gravitational forces may assume one of several possible equilibrium configurations. It can occur that only one configuration is possible; that happens, for example, in a vertical homogeneous cylindrical container in a gravity field that is either absent or directed downward into the liquid. Whenever the contact curve lies entirely on the cylindrical walls, the free surface is completely determined by the contact angle and liquid volume; this is the case for cylinders of arbitrary cross section. However, examples are easily given of containers for which two or more distinct equilibrium configurations can be found.

In earlier papers[1,2] we discuss "exotic" rotationally symmetric containers that have the interesting property of admitting an entire continuum of distinct, rotationally symmetric equilibrium capillary free surfaces, all enclosing the same liquid volume and having the same mechanical energy and contact angle. These families of symmetric free surfaces have the further property that they are unstable, in that certain asymmetric deformations yield surfaces with lower energy.[2,3] In fact, the containers can be so designed that the only energy-minimizing liquid configurations they admit are not symmetric. Although the theory predicts exotic containers for any gravity field, only for microgravity conditions would the length scale be adequately large for accurate physical observation and measurement.

In Ref. 1 these exotic container shapes are calculated by numerical integration of the governing system of nested differential equations, which arise from the classical Young-Laplace equations for equilibrium capillary free surfaces and the condition of prescribed enclosed fluid volume. The shapes are depicted graphically there for a range of gravity accelerations and contact angles of physical interest, along with members of the symmetric equilibrium free-surface continua.

Received Nov. 7, 1990; presented as Paper 91-0107 at the AIAA 29th Aerospace Sciences Meeting, Reno, NV, Jan. 7–10, 1991; revision received May 1, 1991; accepted for publication May 2, 1991.

*Senior Scientist, Lawrence Berkeley Laboratory and Adjunct Professor, Department of Mathematics.
†Professor, Department of Mathematics.
‡Aerospace Engineer, Space Experiments Division.

Here we report on ground-based drop-tower experiments that have been carried out for a particular exotic container, whose radial section is depicted in Fig. 1. This container is for zero gravity and for a contact angle of 80 deg. The container meridian is shown as the solid curve, and the dashed curves (circular arcs for zero gravity) are meridians of some members of the corresponding continuum of symmetric equilibrium free surfaces. The "exotic," bulge portion of the container has been joined top and bottom for this example by right circular cylindrical extensions and disk ends.

Of particular interest for experiments is the property noted above, that a configuration of lower mechanical energy can be obtained by an asymmetric perturbation of the planar member of the family of solution surfaces. Thus, under the idealized Young-Laplace equilibrium contact-angle conditions, if surface friction effects were absent, the family of symmetric equilibrium free surfaces indicated in Fig. 1 would not be observed physically in the container. Through physical experiments we hope to gain an understanding of how a real fluid behaves and to which asymmetric configurations, if any, the fluid might move. The drop-tower experiments reported here are preliminary studies in preparation for a space experiment on the NASA United States Microgravity Laboratory flight (USML-1) scheduled for 1992. Both the theory and the experiments suggest potential application in a number of directions. As one such application we mention the design of containers (such as fuel tanks) for storing liquids under low-gravity conditions.

Container Profile Calculation

Consider the free surface of a liquid that partly fills a rotationally symmetric container oriented with its axis of symmetry parallel to a uniform downward-acting gravitational field, if present. Then a rotationally symmetric equilibrium free surface of the liquid satisfies

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r \sin\psi) = Bu + \lambda \qquad (1)$$

where r is the radial coordinate, u is the height of the surface, ψ is the angle between the horizontal and a meridian of the surface, B is the Bond number ($B \geq 0$), and λ is a parameter that is determined by the geometry and volume constraint.[4] The Bond number is a dimensionless parameter representing the ratio of gravitational to surface forces, and Eq. (1) is the form to which the Laplace capillary equation reduces for rotationally symmetric surfaces. The free surface is to meet the container in a prescribed contact angle γ, $0 < \gamma < \pi$, measured within the liquid (Fig. 2).

The container shapes are determined from the requirement that there be a continuum of rotationally symmetric interfaces obtained from the solutions of Eq. (1), all having the same contact angle, and enclosing the same volume of liquid with the container. If the container meridian is described parametrically by $r = R(\phi)$, $z = Z(\phi)$, where ϕ is the angle with the horizontal (Fig. 2), then R and Z satisfy

$$\frac{dR}{d\phi} = \frac{\cos\phi}{k_c}$$
$$\frac{dZ}{d\phi} = \frac{\sin\phi}{k_c} \qquad (2)$$

Here k_c is the meridional curvature of the container and can be given as an expression involving ϕ, γ, and quantities relating to the solutions of Eq. (1).[1] As shown in our earlier paper,[1] the system (2), with appropriate initial conditions, can be integrated numerically to obtain the container shapes. The resulting shape calculated for $B = 0$, $\gamma = 80$ deg is the curved bulge shown in Fig. 1. The upper and lower circular cylindrical extensions in Fig. 1 are of the same radius, corresponding to the lower extension meeting the bulge in a right angle (i.e., where $\phi = 0$). The upper extension intersects the bulge where $\phi \approx 176$ deg. The 80 deg contact angle corresponds to the materials used for the experiment, acrylic plastic and distilled water.

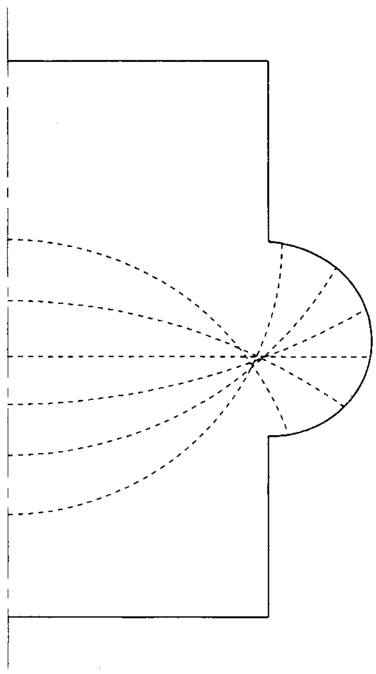

Fig. 1 Radial section of an exotic container for contact angle 80 deg and zero gravity, depicting meridians (*dashed curves*) of members of the symmetric equilibrium free surface continuum.

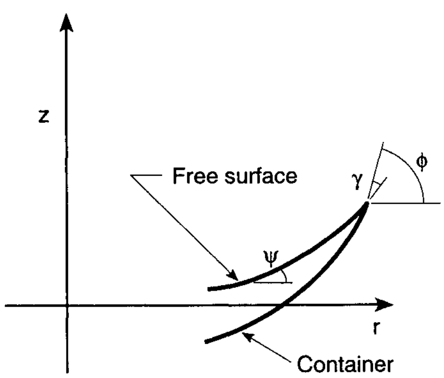

Fig. 2 Coordinates for container and free surface calculations.

Experimental Apparatus

The fill apparatus and vessel are depicted schematically in Fig. 3. The fill apparatus consists of a gravity feed tank, a solenoid valve, and a vapor return line. The vessel interior was bored out of a solid rectangular block of acrylic plastic, to limit optical distortion. The block had a cross section of 10 × 10 cm. The coordinates for the bulged-portion of the cylinder, calculated for a contact angle of 80 deg and scaled to have a maximum bulge diameter of 7.96 cm, were fed into a numerically controlled air-bearing lathe, which performed the final machining operations. As shown in Fig. 1, the circular cylindrical portions bored out above and below the bulge were the same diameter (5.70 cm), meeting the bottom of the bulge in a right angle. The total vessel length was approximately 12.9 cm, of which the height of the bulge was approximately 2.15 cm. All surfaces were lightly finished with cloth and a polishing compound. The interior surface after polishing and annealing deviated less than 50 μm from the specified, calculated one. The vessel, its top and bottom stainless steel flanges, and other components were assembled and installed on an experimental platform suitable for the 5-s drop tower at NASA Lewis Research Center.

Experimental Procedure

The vessel was annealed between each drop-tower run to protect against crazing of the optical surfaces from residual/concentrated stresses caused by the machining and cleaning procedures and rapid deceleration at the bottom of the drop tower. The internal surfaces of the vessel were prepared by sequential rinses with a strong ethanol/distilled water solution and distilled water. The vessel was then allowed to air-dry in a clean-room environment.

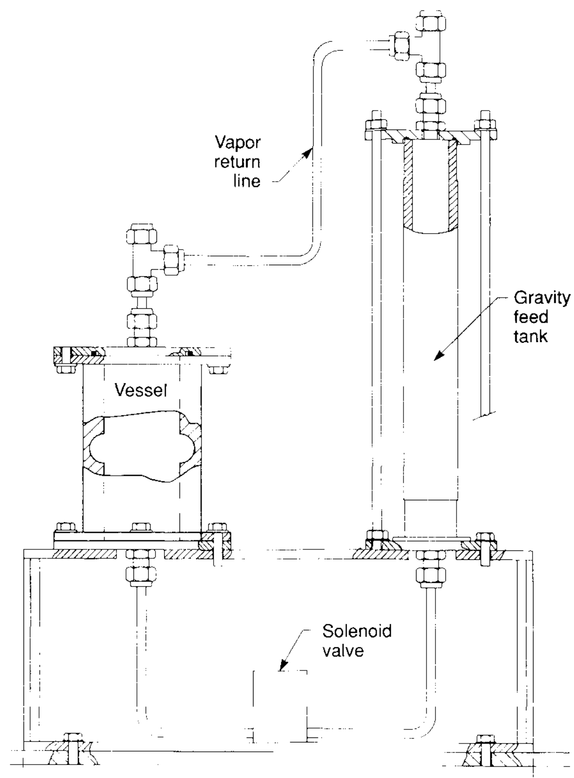

Fig. 3 Schematic of experimental apparatus.

Several procedures to determine the equilibrium contact angle of water on acrylic plastic were performed with specimens prepared in the above manner. These included the standard sessile-drop-tilt/slide method, a capillary tube method, and a wedge method. The first method placed a bound of 58–100 deg on the desired value by indicating the range of hysteresis for the fluid/solid pair. The capillary tube method[5] was applied by machining and polishing a rather large capillary tube (2 cm in diameter), partly filling it with water, and dropping it in the NASA Lewis 2.2-s drop tower. The curvature of the equilibrium free surface (a spherical cap in zero gravity) was measured, and the results were used to determine the contact angle. The value of the contact angle determined by this technique depended significantly on the initial free-surface shape, which could be varied, within the bounds of hysteresis, by small modifications of the fill procedure. The method gave values of between 67 and 90 deg. The wedge method (Ref. 6, pp. 220–221) was found to give the most consistent results. It is based on the discontinuous behavior of a capillary free surface in a wedge. If α is in the interior half-angle of the wedge, then the surface has distinctly different behavior depending on whether the contact angle γ is less than or greater than a critical value depending on α. For a wetting liquid, if $\alpha + \gamma < 90$ deg, fluid climbs up the corner; if $\alpha + \gamma \geq 90$ deg, it does not. In the present case α was varied by adjusting the angle between two contacting vertical plates of acrylic plastic resting in a pool of distilled water. The angle α was decreased until the critical behavior of the fluid was observed. At this point α was measured and γ calculated as 90 deg $- \alpha$. The technique produced contact-angle values of 80 deg for distilled water on acrylic, and the measurement was repeatable to ± 2 deg. These values are consistent with those reported in Ref. 6. A low-gravity refinement of the wedge method (see Ref. 7, pp. 191–192), including possible effects of hysteresis, is being tested with ground-based experiments and will be investigated in a space experiment on the International Microgravity Laboratory IML-2 space flight scheduled for 1994.

The vessel, positioned with its symmetry axis vertical, was filled in the laboratory with the prescribed liquid volume corresponding to a horizontal planar interface, which makes an 80-deg contact angle with the container bulge. For this volume, as predicted, there was no observable curvature in the meniscus; the interface was flat. The fluid was then drawn slowly into the gravity feed tank and the solenoid valve was closed. The experimental platform containing the apparatus was then positioned in the drop tower, which was then sealed and evacuated. Prior to the release of the package the solenoid valve was actuated, permitting the fluid gradually to fill the vessel. This procedure was adopted to keep to a minimum the disturbances introduced to the large and sensitive free surface in filling the container. During the drop, the vessel was backlit by a diffuse white light panel and filmed with a high-speed motion picture camera at 400 frames per second. The film image was subject to any optical distortion resulting from the mismatch in refractive index 1.33 of the distilled water and 1.49 of the acrylic plastic.

Experimental Results

Several experiments were performed for observing the subsequent behavior of the initial 1-g equilibrium interface over the 5-s period of free-fall after release of the vessel in the drop tower. The experiment of central interest was for the vessel filled with the prescribed liquid volume for an initially flat interface, as described above. The flat interface is an equilibrium solution for any gravity level, but for zero gravity the interface is unstable in this vessel under the idealized theory, as are all members of the symmetric equilibrium family depicted in Fig. 1. This experiment was carried out twice. For one case the surface remained essentially flat, in its initial configuration (Fig. 4a). For the other case the surface reoriented to a nonsymmetric one, as shown in Fig. 4b. Which situation occurred probably depended on the nature of the small perturbation imparted to the fluid at the initiation of free-fall. It is significant, however, that for one case the fluid did move in the 5 s of free-fall from its initial flat equilibrium configuration to an obviously nonsymmetric one, in accordance with the mathematical theory.

Two experiments were carried out with the vessel tilted a small amount from vertical, in an attempt to overcome surface friction (resistance at the contact line) that might be preventing reorientation. This procedure biased the surface toward a particular nonsymmetric orientation from the outset. For the experiment with an initial tilt of 2.5 deg, reorientation did not occur, whereas for a 5-deg initial tilt, a shape like that depicted in Fig. 4b resulted. Thus, of the total of four experiments performed with initially horizontal planar or tilted interfaces, two reoriented to an asymmetric configuration like the one depicted in Fig. 4b.

Additional experiments were carried out, investigating the effect of varying the fluid volume from that corresponding to the planar interface fill level. The horizontal planar interface fill volume is 314 ml, of which the portion of the fluid in the bulge is approximately 48 ml. The amounts of fluid less than, or in excess of, the planar interface fill level were generally about 10 ml. For an initially vertical vessel, the one underfilled case resulted in reorientation during free-fall to a symmetric surface that domed upward in the center. The over-filled cases were tested on initially tilted vessels; of these, two with initial

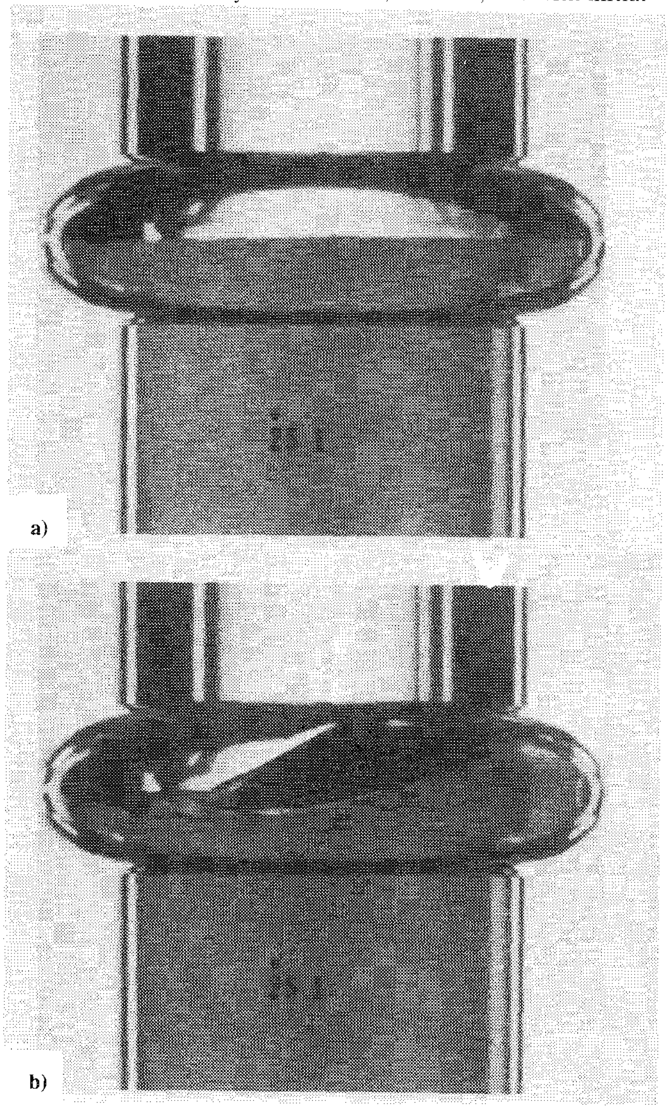

Fig. 4 Fluid configurations: a) flat interface initial configuration; b) non-symmetric terminal configuration.

tilts of 2.5 deg and 5 deg reoriented to a surface similar to the one shown in Fig. 4b. For the other, with a 1-deg tilt (and only about a 3-ml overfill), although an apparently stable "domed-down" meniscus was formed, it remained "off-axis," maintaining the initial tilt with respect to the container.

The off-axis "sticking" phenomenon was investigated for similar experiments performed with right circular cylinders for some larger contact-angle fluid/solid pairs (i.e., with wetting angles substantially closer to 90 deg than to 0 deg), such as water and acrylic. (Typically, contact-angle hysteresis for wetting liquids is more prominent at larger contact angles.) The tests were carried out with 2-cm diameter cylinders on the smaller, less controlled, but more readily available 2.2-s drop tower. The containers were tilted initially by amounts between 5 deg and 10 deg. For the smaller tilts the initial surface inclination with respect to the container persisted after the cylinder was introduced into free-fall. For the larger initial tilts, the ensuing motion during free-fall resulted in the contact line becoming "unstuck" and the surface reorienting toward the axially symmetric minimum-energy state predicted by the Young-Laplace theory.

Concluding Remarks

As discussed above, the result of central interest is that in one of the experiments the fluid did, in fact, move away from an initial horizontal flat equilibrium configuration to a nonsymmetric one during the period of free-fall (Figs. 4a and 4b). This is in accordance with the mathematical result that the family of symmetric equilibrium interfaces cannot be stable and that a certain asymmetric deformation of the surface decreases the energy. The effects of surface friction and hysteresis, which are not included in the idealized Young-Laplace formulation on which the mathematical theory is based, were indicated to some extent in the experiments. For the materials used, the motion-inhibiting effects of surface friction could apparently be overcome by the forces resulting from a sufficiently large initial tilt of the container from the vertical. For smaller tilts, or no tilt at all, the circumstance that the fluid reoriented during free-fall in some cases and not in others could likely be attributed to the surface friction and the varying nature of the small disturbances imparted to the fluid at the initiation of free-fall.

For the overfilled and underfilled cases the containers do not satisfy the criterion of being exotic, since the continuum of equilibrium symmetric surfaces, as depicted in Fig. 1, is not possible for these fill volumes. Although there are apparently lower energy asymmetric configurations, as suggested by two of the initially tilted overfilled cases, the path to them from an initial configuration might be more tenuous without the intervening continuum of equi-energy surfaces. More complete determination of possible energy-minimizing equilibrium configurations and the role of surface friction "sticking" await the controlled, longer term experiments possible in space.

Acknowledgments

This work was supported in part by NASA under Grant NAG3-1143, by the National Science Foundation under Grant DMS89-02831, and by the Mathematical Sciences Subprogram of the Office of Energy Research, U.S. Department of Energy, under Contract Number DE-AC03-76SF00098.

References

[1]Concus, P., and Finn, R., "Exotic Containers for Capillary Surfaces," *Journal of Fluid Mechanics*, Vol. 224, 1991, pp. 383–394; Corrigenda, *Journal of Fluid Mechanics* (to be published).

[2]Finn, R., "Nonuniqueness and Uniqueness of Capillary Surfaces," *Manuscripta Mathematica*, Vol. 61, 1988, pp. 347–372.

[3]Concus, P., and Finn, R., "Instability of Certain Capillary Surfaces," *Manuscripta Mathematica*, Vol. 63, 1989, pp. 209–213.

[4]Finn, R., *Equilibrium Capillary Surfaces*, Springer-Verlag, New York, 1986.

[5]Chai, A., "Accurate Measurement of Contact Angles," AIAA 27th Aerospace Sciences Meeting, Reno, NV, Jan. 1989.

[6]Concus, P., and Finn, R., "On Capillary Free Surfaces in a Gravitational Field," *Acta Mathematica*, Vol. 132, 1974, pp. 207–223.

[7]Concus, P., and Finn, R., "Capillary Surfaces in Microgravity," *Low-Gravity Fluid Dynamics and Transport Phenomena*, edited by J. N. Koster and R. L. Sani, Progress in Astronautics and Aeronautics, Vol. 130, AIAA, Washington, DC, 1990, pp. 183–206.